## Appendix Learning From Mistakes: Machine Learning Enhanced Human Expert Effort Estimates

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In this appendix we explain the mathematical formulation of the Linear Programming model we used to predict the MisestimationMagnitude (RQ2) in our paper [1].<sup>1</sup>

Linear Programming (LP) [2] aims to achieve the best outcome from a mathematical model with a linear objective function subject to linear equality and inequality constraints. The feasible region is given by the intersection of the constraints and the Simplex (linear programming algorithm) is able to find a point in the polyhedron where the function  $X_i > 0$  has the smallest value (minimisation) in polynomial time.

Here, we generalize the model proposed for the effort estimation by Sarro and Petrozziello [3]. In the original implementation, the model is subject to an inequality constraint imposing that the value estimated for each of the  $X_i < 0$ observations in the training set has to fall in  $R_0^+$ . Here, we remove the inequality constraints allowing the model to use both positive and negative feature values as well as to optimize for both positive and negative values, as follows:

minimise 
$$\sum_{i=1}^{n} |\sum_{j=1}^{m} a_{ij}x_j - ActualValue_i|$$

$$x_j \text{free}, \qquad j = 1, ..., m$$
(1)

where  $a_{ij}$  represents the coefficient of the  $j^{th}$  feature for the  $i^{th}$  project,  $x_j$  is the value of the  $j^{th}$  feature, and *ActualValue*<sub>i</sub> is the actual effort of the  $i^{th}$  project.

Due to the non-linearity of the absolute value function, the above model has been linearised as follows:

$$\begin{array}{ll} \text{minimise} & \sum_{i=1}^{n} t_{i} \\ \text{subject to} & \sum_{i=1}^{n} \sum_{j=1}^{m} a_{ij} x_{j} - Actual Value_{i} - t_{i} \leq 0 \\ & \sum_{i=1}^{n} \sum_{j=1}^{m} a_{ij} x_{j} - Actual Value_{i} + t_{i} \geq 0 \\ & x_{j} \text{ free,} \\ & t_{i} \text{ free,} \\ \end{array}$$

Let  $X_i$ ,  $\forall i$  be the part of Eq. (1) wrapped in the absolute value.  $\forall i$ , the slack variable  $t_i$  and the following two constraints have been added to the model:

$$X_i \le t_i$$

 $-X_i \leq t_i$ 

Therefore we can have one of the following cases:

- $T_i > 0$  The second constraint,  $-X_i \le t_i$ , is always fulfilled as  $-X_i$  is negative and  $t_i$  is implicitly  $\ge 0$ . Since  $t_i$  is minimised by the objective function and  $0 \le X_i \le t_i$ , the first constraint,  $X_i \le t_i$ , is satisfied and  $t_i$  is abs(X).
- $X_i < 0$  The first constraint,  $X_i \le t_i$ , is always fulfilled as  $X_i$  is negative and  $t_i$  is implicitly  $\ge 0$ . Since  $t_i$  is minimised by the objective function and  $0 \le -X_i \le t_i$ , the second constraint,  $-X_i \le t_i$ , is satisfied and  $t_i$  is abs(X).
- $X_i = 0$  Both constraints are always fulfilled since  $t_i$  is implicitly  $\geq 0$ . Since  $t_i$  is minimised by the objective function,  $0 = X_i = t_i$ . So  $t_i$  is abs(X).

## REFERENCES

- F. Sarro, R. Moussa, A. Petrozziello, and M. Harman, "Learning from mistakes: Machine learning enhanced human expert effort estimates," *IEEE Transactions on Software Engineering*.
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